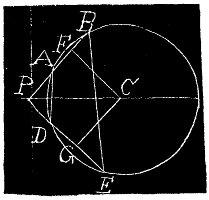
$$\therefore\ \triangle = aR\int_{\theta''}^{\theta'}[\sqrt{(1-e^2\sin^2\theta)}\sin\theta+\sqrt{(1-e^2\cos^2\theta)}\cos\theta]d\theta\Big/\int_{\theta''}^{\theta'}d\theta$$

$$=\frac{aR}{4\sin^{-1}(R/a)-\pi}\Big[\sin\theta'\sqrt{(1-e^2\cos^2\theta')}$$

$$-\sin\theta''\sqrt{(1-e^2\cos^2\theta'')}-\cos\theta'\sqrt{(1-e^2\sin^2\theta')}$$

$$+\cos\theta''\sqrt{(1-e^2\sin^2\theta'')}$$



$$+\frac{1-e^2}{e}\log\left(\frac{e\sin\theta'+\sqrt{(1-e^2\cos^2\theta')}}{e\sin\theta''+\sqrt{(1-e^2\cos^2\theta'')}}\right)$$

$$-\frac{1-e^2}{e}\log\left(\frac{e\cos\theta'+\sqrt{(1-e^2\sin^2\theta')}}{e\cos\theta''+\sqrt{(1-e^2\sin^2\theta'')}}\right)\Big].$$

But $\sin\theta'=\cos\theta''=R/a$, $\sin\theta''=\cos\theta'=[\sqrt{(a^2-R^2)}]/a$.
$e^2\sin^2\theta'=e^2\cos^2\theta''=1$, $e^2\sin^2\theta''=e^2\cos^2\theta'=[(a^2-R^2)/R^2]=e^2-1$.
Whence by substitution and reduction we get

$$\triangle=\frac{R^2}{2\sin^{-1}(R/a)-\frac{1}{2}\pi}\left[\sqrt{(2-e^2)}+(1-e^2)\log\left(\frac{1+\sqrt{(2-e^2)}}{\sqrt{(e^2-1)}}\right)\right].$$

83. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy, Irving College, Mechanicsburg, Pa.

Find the average area of all ellipses whose semi-axis major is a.

I. Solution by J. W. YOUNG, Fellow and Assistant, Ohio State University, Columbus, Ohio.

The area of an ellipse whose major-axis is a, and whose minor-axis is b, is πab. We must find the average of all possible values of this expression as b varies from zero to a.

$$\therefore \text{Average required}=\frac{\pi a\int_0^a bdb}{\int_0^a db}=\tfrac{1}{2}\pi a^2,$$

$=\frac{1}{2}$ the area of the circle whose radius is the major-axis of the ellipse.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

1. Let x=semi-conjugate axis. Then average area

$$=\pi a\frac{\int_0^a xdx}{\int_0^a dx}=\tfrac{1}{2}\pi a^2=1.5708a^2.$$

2. Let e=eccentricity. Then area$=\pi a^2\sqrt{(1-e^2)}$.

$$\therefore \text{ Average area}=\pi a^2 \frac{\int_0^1 \sqrt{(1-e^2)}}{\int_0^1 de}=\tfrac{1}{4}\pi^2 a^2=2.4674a^2.$$

MISCELLANEOUS.

73. Proposed by CHAS. E. MYERS, Canton, Ohio.

In an ice cream freezer, cream of a homogeneous character and at the uniform temperature of 60° Fahrenheit is put into a cylinder having a closed base, and the whole put into a freezing mixture so as to subject the base and convex surface to a constant temperature of 30° Fahrenheit. Required the temperature at any point within the cream after the expiration of a given time. [From *Higher Mathematics*.]

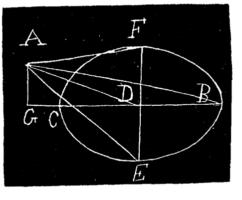
No solution of this problem has been received.

74. Proposed by S. HART WRIGHT, M. D., A. M., Ph. D., Penn Yan, N. Y.

The longest diameter of a horizontal ellipse is $CB=2a=6$ feet. Its shortest diameter is $EF=2b=4$ feet, their intersection being at D. Find in an indefinite vertical plane passing through CB, a point $A=5$ feet$=c$ from D, the ellipse being seen from A as a circle.

I. Solution by the late B. F. BURLESON, and the PROPOSER.

The eye being at A, and the ellipse being projected as a circle, CB and EF subtend equal angles at A, or $\angle EAF=\angle BAC$. Produce DC to G, A being vertically over G, and put $CG=x$, and $GA=y$, and $\angle ADC=\phi$=angle of elevation of A.



Then $y=\sqrt{[c^2-(a+x)^2]}$ ......(1).

$AB=\sqrt{[(2a+x)^2+y^2]}$ .....(α).

$\sin\angle ACG=\sin\angle ACB=y/\sqrt{(x^2+y^2)}$......($\beta$), and $\tan\angle EAD=b/c$ .....(γ).

$\therefore \sin\angle EAF=\sin\angle BAC=2dc/(b^2+c^2)$...... ($\delta$).

From $\triangle BAC$ we have the proportion, $AB : \sin\angle ACB :: BC : \sin\angle BAC$.

$$\therefore \frac{2bc\sqrt{[(2a+x)^2+y^2]}}{b^2+c^2}=\frac{2ay}{\sqrt{(x^2+y^2)}}......(2).$$

Resolving (1) and (2) we have $x=\dfrac{c\sqrt{[(c^4-a^2b^2)(a^2-b^2)]}-a}{a(c^2-b^2)}$

$$=\tfrac{5}{63}\sqrt{(2945)}-3=1.30697255 \text{ feet}.$$

$$\therefore y=\frac{bc(c^2-a^2)}{a(c^2-b^2)}=2\tfrac{34}{63} \text{ feet}.$$